The Mikheyev-Smirnov-Wolfenstein effect as a probe of the solar interior

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We relate the MSW effect to the efective absorption of the electronic collective motion energy by retaining the imaginary part of the index of refraction associated with the charged-current scattering and show that the small angle MSW solution to the solar neutrino anomaly can be used as a probe of the physical conditions of the solar interior if it is correct. We find that the constraint on the absorption imposed by the small angle MSW solution and the theoretical estimate of the absorption by the Boltzmann kinetic theory are consistent, which shows that a consistent theoretical picture can be developed when plasma absorption processes are taken into account.

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If, based on its apparent success, the small angle MSW solution [1–3] is accept as the solution for the solar neutrino problem [4–9], it can be used as a probe of physical conditions in the solar interior. Since the MSW effect [10] can be attributed to the resonant scattering of neutrinos off electrons in the solar interior, it must be sensitive to the absorption of not only the neutrino energy but also the electron energy. The effective absorption can be represented in terms of complex indices of refraction. In order to take into account the absorption, we rewrite the index of refraction n_e associated with the charged-current scattering in such a suggestive form [10]:

$$n_e = 1 + \frac{G_F}{4\pi E\alpha\lambda_c}\omega_e^2, \tag{1}$$

where G_F, $\alpha = e^2$, $\lambda_c = m_e^{-1}$, E and ω_e are, respectively, the Fermi constant, fine structure constant, electron Compton wavelength, neutrino energy and electronic plasmon energy, noticing that we have set $\hbar = c = 1$. Since the MSW effect depends on an effective density-dependent contribution to the neutrino mass, longitudinal plasmons or Langmuir waves due to electrostatic oscillations of free electrons in the solar plasma make sense [11–13]:

$$\omega_e = (\omega_{pe}^2 + 3V_{Te}^2 k'^2)^{1/2} + i\Gamma', \tag{2}$$

where $\omega_{pe} = (4\pi N_e e^2/m_e)^{1/2}$, $V_{Te} = (T/m_e)^{1/2}$, k' and Γ' are, respectively, the electronic plasma frequency, characteristic thermal velocity of electrons, plasmon momentum (or wavenumber) and effective absorption coefficient of the plasmon. The fact that ω_e is complex means that the plasmon or the coherent motion of electrons in the plasma $e^{-i\omega_e t}$ will decay. If the plasma is not uniform, all dependent variables will depend on the location. Obviously, if $k' = 0$ and $\Gamma' = 0$, then $\omega_e = \omega_{pe}$, and hence $E(n_e - 1) = G_F N_e$, as assumed in the MSW theory [10,1–3]. The purpose of this letter is to investigate how the small angle MSW solution constrains Γ' in the solar plasma.

For simplicity, we consider two neutrino flavors. If we use the flavor eigenstates $|\nu_e\rangle$ and $|\nu_\mu\rangle$ as the basis, the time evolution of the neutrino state vector $|\nu(x)\rangle = a_e(x)|\nu_e\rangle + a_\mu(x)|\nu_\mu\rangle$ in matter in the relativistic limit is governed by the equation [10,3]

$$i\frac{d}{dx}\begin{pmatrix} a_e \\ a_\mu \end{pmatrix} = \frac{\Delta_V}{2}\begin{pmatrix} M(x) & \sin 2\theta_V \\ \sin 2\theta_V & -M(x) \end{pmatrix}\begin{pmatrix} a_e \\ a_\mu \end{pmatrix} \tag{3}$$

where $x = ct$, $M(x) = \sqrt{2}E(n_e(x) - 1)/\Delta_V - \cos 2\theta_V$, $\delta m^2 = m_2^2 - m_1^2$, $\Delta_V = \delta m^2/2E$, θ_V is the vacuum mixing angle. $M_r(x) = \mathrm{Re}\, M(x)$ and $M_i(x) = \mathrm{Im}\, M(x)$ can be approximately cast as follows:

$$\begin{aligned} M_r(x) \approx{}& \sqrt{2}G_F N_e(x)(1 + 3k^2 - \Gamma(x)^2)/\Delta_V \\ & - \cos 2\theta_V, \end{aligned} \tag{4a}$$

$$M_i(x) \approx 2\sqrt{2}G_F N_e(x)\Gamma(x)/\Delta_V, \tag{4b}$$

where $k = k'/k_D$, $\Gamma(x) = \Gamma'(x)/\omega_{pe}(x)$, and $k_D = \omega_{pe}/V_{Te}$. We have assumed $k^2 \ll 1$, which is reasonable because when $k \gtrsim 0.3$ the Landau damping [11–14] will cut in. The fact that $M(x)$ is complex implies that there is no $N_e(x_c)$ so that $M(x_c) = 0$ no matter $N_e(x)$ is large or small. Novertheless, if the absorption Γ is small enough, the level crossing still occurs when

$$M_r(x_c) \geq 0. \tag{5}$$

Obviously, k favors resonance, while Γ disfavors resonance as it should.

At a free electron number density, $N_e(x)$, the light (L) and heavy (H) local mass eigenstates [15,3] are

$$|\nu_L(x)\rangle = \cos\theta(x)|\nu_e\rangle - \sin\theta(x)|\nu_\mu\rangle \tag{6a}$$

$$|\nu_H(x)\rangle = \sin\theta(x)|\nu_e\rangle + \cos\theta(x)|\nu_\mu\rangle, \tag{6b}$$

which have complex eigenvalues $\pm\frac{1}{2}\Delta(x)$, where

$$\Delta(x) = \Delta_V[M(x)^2 + \sin^2 2\theta_V]^{1/2}, \tag{7}$$

and $\theta(x)$ satisfies

$$\sin 2\theta(x) = \frac{\sin 2\theta_V}{\Delta(x)/\Delta_V}, \tag{8a}$$

$$\cos 2\theta(x) = \frac{-M(x)}{\Delta(x)/\Delta_V}. \tag{8b}$$

Since

$$\Delta(x_c) = \Delta_V(\sin^2 2\theta_V - 4\Gamma_c^2 \cos^2 2\theta_V)^{1/2}, \tag{9}$$

the absorption will affect the width of the resonance: $2\Delta(x_c)$, where $\Gamma_c = \Gamma(x_c)$. The corresponding resonance distance [3] is

$$\delta x = 2\left[-\frac{\dot{N}_e}{N_e}\right]_{x_c}^{-1} (\tan^2 2\theta_V - 4\Gamma^2)^{1/2}, \tag{10}$$

where $\dot{N}_e(x) = dN_e(x)/dx$ is the electronic density gradient. Because the small angle MSW solution gives the best fit to existing solar neutrino data, $\sin^2 2\theta_V \approx 8\times 10^{-3}$, and $\delta x \geq 0$, we obtain

$$|\Gamma_c| \leq \tfrac{1}{2}\tan 2\theta_V \approx 0.04, \tag{11}$$

which is the upper limit of $|\Gamma_c|$ in the solar interior where the resonance takes place.

The fact that the eigenvalues are complex implies that the neutrino states will decay, too. In order to see this we express the neutrino state vector $|\nu(x)\rangle$ in the matter in terms of the local mass eigenstates $|\nu(x)\rangle = a_H(x)|\nu_H(x)\rangle + a_L(x)|\nu_L(x)\rangle$. Consequently, the evolution equation becomes

$$i\frac{d}{dx}\begin{pmatrix} a_H \\ a_L \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\Delta(x) & i\alpha(x) \\ -i\alpha(x) & -\frac{1}{2}\Delta(x) \end{pmatrix}\begin{pmatrix} a_H \\ a_L \end{pmatrix}, \tag{12}$$

where

$$\alpha(x) \approx \frac{\Delta_V}{2}\frac{\sqrt{2}G_F\dot{N}_e(x)\sin 2\theta_V}{\Delta(x)^2}$$

enforces mixing of the mass eigenstates governed by the density gradient, where we have assumed $d\Gamma/dx = 0$. When the off-diagonal elements can be neglected with respect to the diagonal elements, i.e., when

$$\gamma(x) = \left|\frac{\Delta(x)}{\alpha(x)}\right| \approx \frac{\sin^2 2\theta_V}{\cos 2\theta_V}\frac{|\Delta(x)^3|}{\Delta_V^2\sin^3 2\theta_V}\left|\frac{\dot{N}_e(x)}{N_c}\right|^{-1} \gg 1,$$

the neutrino will propagate adiabatically through the matter. The absorption tends to weaken this condition, for example, near the crossing point, it reads

$$\gamma_c = \gamma_c^0(1 - 4\Gamma_c^2\cot^2 2\theta_V)^{3/2} < \gamma_c^0. \tag{13}$$

where $\gamma_c^0 = \gamma(x_c; \Gamma_c = 0)$. Since the detected solar ^{7}Be neutrino flux [5–9] is much less than the predicted [4], the ^{7}Be neutrinos must propagate adiabatically in the sun, the absorption correction factor thus should not depart from unity significantly, which demands

$$|\Gamma_c| \ll 0.04. \tag{14}$$

Of cause, the absorption also affects the adiabatic boundary [15] of $\gamma_c \sim 1$ in the $\delta m^2/E - \sin^2 2\theta_V$ plane.

When $\gamma_c \gg 1$, the adiabatic states are

$$|L\rangle = e^{-ia(x_0;\,x)+b(x_0;\,x)}|\nu_L(x)\rangle, \tag{15a}$$
$$|H\rangle = e^{ia(x_0;\,x)-b(x_0;\,x)}|\nu_H(x)\rangle, \tag{15b}$$

where

$$a(x_0;\,x) = \tfrac{1}{2}\int_{x_0}^{x}\Delta_r(x')dx', \tag{16a}$$
$$b(x_0;\,x) = \tfrac{1}{2}\int_{x_0}^{x}\Delta_i(x')dx', \tag{16b}$$

where x_0 is the production point of the basis states, $\Delta_r(x) = \mathrm{Re}\,\Delta(x)$, $\Delta_i(x) = \mathrm{Im}\,\Delta(x)$. Obviously, they will decay provided that $b(x) \neq 0$. If $b(x) > 0$, the local heavy mass eigenstate will continuously jump down into the local light mass eigenstate, while the local light mass eigenstate will continuously jump up into the local heavy mass eigenstate; if $b(x) < 0$ the processes are reversed. Expressions (7), (8b), (16b) and condition (14) show that $b(x) < 0$ if both the plasmons are indeed absorbed ($\Gamma < 0$) and the plasma density is supercritical ($M_r > 0$) or both the plasmons are excited ($\Gamma > 0$) and the plasma density is subcritical ($M_r < 0$). However, it is still convenient to use these states as the basis states in the region for which there are no transitions [3]. Under the adiabatic approximation, we have $|\nu(x)\rangle = a_1|L\rangle + a_2|H\rangle$ in which the linear combination coefficients a_i (i=1, 2) are determined by the initial condition, hence the average probability of detecting an electron neutrino at the Earth with the initial condition $|\nu(x_0)\rangle = |\nu_e\rangle$ is

$$P_{\nu_e}^{ad} = \tfrac{1}{2}(1 + \cos 2\theta_0\cos 2\theta_V)\cosh 2b_V + \tfrac{1}{2}(\cos 2\theta_0 + \cos 2\theta_V)\sinh 2b_V, \tag{17}$$

where $\theta_0 = \theta(x_0; \Gamma = 0)$, $b_V = b(x_0; R_\odot)$. This expression will reduce to the well-known adiabatic probability if $\Gamma \equiv 0$ or $b_V = 0$, noticing that $\sinh 2b_V = 0$ and $\cosh 2b_V = 1$ when $b_V = 0$. This shows that the absorption also affects the detection probability.

However, it is necessary to go beyond the adiabatic approximation because the solar pp and ^{8}B neutrinos are suppressed partially [1–9]. We follow Parke [3] to do so. Eqs. (10), (13) and (17) show that the critical region for nonadiabatic behavior occurs in a narrow region $[x_-,\, x_+]$ (for small θ_V) surrounding the crossing point x_c and that this behavior is controlled by both the density gradient $[\dot{N}_e/N_e]_{x_c}$ and the absorption Γ_c at the crossing point, where $x_\pm = x_c \pm \frac{1}{2}\delta x$. See Fig. 1 of Bethe [3] for graphical understanding. When an initial electron neutrino at x_0 approaches to the inner boundary of the nonadiabatic region, its state becomes

$$|\nu(x_-)\rangle = \cos\theta_0 e^{-ia_- + b_-}|\nu_L(x_-)\rangle + \sin\theta_0 e^{ia_- - b_-}|\nu_H(x_-)\rangle, \tag{18}$$

where $a_- = a(x_0; x_-)$, $b_- = b(x_0; x_-)$. As the neutrino goes through the nonadiabatic region to reach its outer boundary x_+, we have the mixed states according to Parke [3]

$$|\nu_L(x_-)\rangle \to a_1|\nu_L(x_+)\rangle + a_2|\nu_H(x_+)\rangle, \tag{19a}$$
$$|\nu_H(x_-)\rangle \to -a_2^*|\nu_L(x_+)\rangle + a_1^*|\nu_H(x_+)\rangle, \tag{19b}$$

where a_i (i=1, 2) are determined by the nature of the transition point and satisfy $|a_1|^2 + |a_2|^2 = 1$. From the upper boundary on, the eigenstates $|\nu_L(x)\rangle$ and $|\nu_H(x)\rangle$ will evolve adiabatically starting with $|\nu_L(x_+)\rangle$ and $|\nu_H(x_+)\rangle$ respectively. Therefore, the state vector of the neutrino reads as follows:

$$|\nu(x)\rangle = A|\nu_L(x)\rangle + B|\nu_H(x)\rangle, \tag{20}$$

in the detection region $x \geq x_c$, where

$$A = a_1 \cos\theta_0 e^{-iA_+ + B_+} - a_2^* \sin\theta_0 e^{-iA_- + B_-} \tag{21a}$$
$$B = a_2 \cos\theta_0 e^{iA_- - B_-} + a_1^* \sin\theta_0 e^{iA_+ - B_+} \tag{21b}$$

in which $A_\pm = a_+ \pm a_-$, $B_\pm = b_+ \pm b_-$, $a_+ = a(x_+; x)$ and $b_+ = b(x_+; x)$.

Substituting Eqs. (6a) and (6b) into Eq. (20), one can find the amplitude for producing, in the solar core x_0, and detecting, on the Earth, an electron neutrino after passage through resonance: $A_e = A\cos\theta_V + B\sin\theta_V$. Thus the probability of detecting this neutrino as an electron neutrino after averaging over both the production and the detection positions is given by

$$\begin{aligned} P_{\nu_e} = {} & |a_1|^2(\cos^2\theta_0\cos^2\theta_V e^{2B_+} + \sin^2\theta_0 \sin^2\theta_V e^{-2B_+}) \\ & + |a_2|^2(\cos^2\theta_0 \sin^2\theta_V e^{2B_-} + \sin^2\theta_0\cos^2\theta_V e^{-2B_-}), \end{aligned} \tag{22}$$

where $b_+ = b(x_+; R_\odot)$. If $B_\pm = 0$, we reproduce the well-known Parke formula [1]:

$$P_{\nu_e}^{Parke} = \tfrac{1}{2} + (\tfrac{1}{2} - P_x)\cos 2\theta_0 \cos 2\theta_V, \tag{23}$$

where $|a_1|^2 = 1 - |a_2|^2$ and $P_x = |a_2|^2$ is the probability of transition from $\nu_H(x_-)\rangle$ to $\nu_L(x_+)\rangle$ (or vice versa) as the neutrino goes through the nonadiabatic region. Parke [3] and Haxton [15] have worked out P_x by using a linear density profile, it is natural to generalize it to our case with $\Gamma \neq 0$:

$$P_x = \exp(-\frac{\pi}{2}\gamma_c). \tag{24}$$

Because the small angle MSW solution with $B_\pm = 0$ gives the best fit to existing solar neutrino data, the solar neutrino experiments require $|B_\pm| \ll 1$. Using the mean density $\rho_\odot \sim 1$ g cm^{-3}, the mean number of electrons per nucleon $Y_e = \frac{1}{2}$, we can estimate $|B_\pm| \sim 10^2|\overline{\Gamma}|$, where $\overline{\Gamma}$ is the mean absorption coefficient in the solar interior. Consequently, the small angle MSW solution demands

$$|\overline{\Gamma}| \ll 10^{-2}. \tag{25}$$

If we assume that the resonance region is near the production region, then $b_- \approx 0$, hence $B_+ \approx B_- \approx b_+$. Since $\Delta_i(x) \approx -\Gamma(x)2\sqrt{2}G_F N_e(x)\cos 2\theta'(x)$, where $\theta'(x) = \theta(x; \Gamma = 0)$, and $\cos 2\theta'(x) > 0$ when $x > x_+$, then $\Delta_i(x) > 0$ when $\Gamma(x) < 0$. Therefore, $B_\pm > 0$ when $\Gamma(x) < 0$, but $B_\pm < 0$ when $\Gamma(x) > 0$. Since numerical evaluation shows that Eqs. (22) and (23) give the same results when $B_\pm \lesssim 10^{-3}$, we may infer that $\Gamma(x > x_c) < 0$ and $|\overline{\Gamma}| \lesssim 10^{-5}$ in the solar interior.

The Boltzmann kinetic theory formulated in the Boltzmann equation [11] can be used to estimate the effective absorption coefficient of the electronic energy. The effective energy transition rate Z_{ei} and momentum transition rate $\mathbf{R}_{ei}$ from electrons to ions per electron in the elastic scattering process $\mathbf{p}_e + \mathbf{p}_i \rightleftharpoons \mathbf{p}'_e + \mathbf{p}'_i$ (Chapter 3 of [11]) can be estimated through Tayler-expanding these rates near local thermodynamic equilibrium between electrons and ions as follows

$$Z_{ei} = -\nu_Z^{ei}(T_e - T_i), \tag{26a}$$
$$\mathbf{R}_{ei} = -\nu_R^{ei}(T_e - T_i)(m_e\mathbf{u}_e - m_i\mathbf{u}_i), \tag{26b}$$

where ν_Z^{ei} and $\Gamma' = -\nu_R^{ei}(T_e - T_i)$ are the effective energy and momentum transition (or absorption) coefficients of electrons in the plasma, $\mathbf{u}_{e(i)}$ is the coherent motion velocity of electrons (ions), noticing that $Z_{ei}^{eq} = 0$ and $\mathbf{R}_{ei}^{eq} = 0$ when electrons and ions have the same temperature $T_e = T_i$. Since the effective energy and momentum transition rates are dependent on each other, $\Gamma' = -\nu_Z^{ei}(T_e - T_i)/T_e$. Therefore, we need only the effective energy transition coefficient ν_Z^{ei}, which may be estimated by the thermal timescale of the sun, the timescale that the stored thermal energy of the sun is used up via radiation at the present radiation power. The thermal timescale is equal to the gravitational timescale, which is the ratio of the gravitational energy to the total luminosity [4]

$$t_{gravity} \sim GM_\odot^2/R_\odot L_\odot \approx 10^7 \,\text{yr}. \tag{27}$$

So we may have

$$\Gamma' \sim -10^{-14}(T_e - T_i)/T_\odot \sec^{-1}. \tag{28}$$

Since $\omega_{pe} \gg 1$ sec^{-1} and $T_e \gtrsim T_i$ in the solar interior, we know $\Gamma = \Gamma'/\omega_{pe}$ is negtive and $|\Gamma| \ll 10^{-5}$ in the solar interior. This shows that the small angle MSW solution to the solar neutrino problem and the Boltzmann kinetic theory are consistent. A treatment of the plasma processes including absorption thus verifies that a consistent theoretical picture can be developed.

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